***Numerical SOLUTIONS***

***to Odd-Numbered Problems****

***of Chapters 1 – 10 of:***

# *Strength and Stiffness of Engineering Systems*


by: Frederick A. Leckie

Dominic J. Dal Bello


## Solutions Version: 2009.1


Solutions by: D. J. Dal Bello

domdalbello@yahoo.com


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**Note*: ***Chapters 3 and 8*** include several multi-part problems – both odd- and even-numbered – for which each part repeats the question of Part (a), but with different numerical values. For such problems (including even-numbered problems), the numerical solutions are given for Parts (a), (c), (e), etc. The problems are: 3.20, 8.1, 8.3, 8.4, 8.13-8.16 and 8.20.

## Chapter 1

**1.1** Ultimate, Proof, Working

**1.3** 79.0 kips

**1.5** $W_{min}$ = 4710 lb, 3-ton (6000-lb) crane

**1.7** 2.5; 64%

**1.9** 6400 lb; 12.8 kips, 11.2 kips, 3200 lb; 18.4 kips; 38.4 kips

**1.11** two 8 kN loads, $R_{Ax}$, $R_{Ay}$, $R_{By}$

## Chapter 2

**2.1** $P_{CD}$ = 2.5 kips; $P_{BC}$ = 1.5 kips; $P_{EF}$ = –3.0 kips; $P_{BE}$ = 2.50 kips

**2.3** $T_2$:$T_1$ = 2:1

**2.5** 300 lb; 2400 lb-ft

**2.7** $M_{max}$ = 41.4 kN·m for $a$ = 4 m or 6 m

**2.9** 2160 lb; 180 lb/ft, 90 lb/ft; 3240 lb-ft

**2.11** $P_{AB}$ = –3.23 kips; $T$ = 16.79 kip-in. CCW; $N$ = 1.120 kips up

## Chapter 3

**3.1** 0.1042%, 31.3 ksi; 0.128 in$^2$; 1.92

**3.3** –4.04 ksi

**3.5** Plot; $S_p$ ~ 225 MPa; $E$ = 200 GPa; $S_y$ = 255 MPa; $S_u$ = 450 MPa

**3.7** 127.3 MPa; 1.88; 1.455 mm; 9.994 mm

**3.9** $-720\times10^{-6}$ in.; 0.029 in$^3$

**3.11** 156.3 kN·m/m$^3$; 312.5 N·m

**3.13** 30,000 cycles; 40%; 40.8 kN

**3.15** 158 MPa; 131 MPa

**3.17** 75.8 ksi

**3.19** 188.5 kN; 94.3 kN, 3.62 kN·m, 1.810 kN·m

**3.20** Strain ($\times10^{-3}$)

| Part | $\varepsilon_x$ | $\varepsilon_y$ | $\varepsilon_z$ | $\gamma_{xy}$ |
|---|---|---|---|---|
| (a) | +2.86 | –0.94 | –0.94 | +2.68 |
| (c) | +2.39 | +0.49 | –1.41 | +2.68 |
| (e) | +2.14 | –2.61 | +0.24 | +3.57 |
| (g) | –2.39 | –0.49 | +1.41 | +3.57 |

**3.21** Strain ($\times10^{-6}$)

| Part | $\varepsilon_x$ | $\varepsilon_y$ | $\varepsilon_z$ |
|---|---|---|---|
| (a) | +350 | –83.3 | +133.3 |
| (b) | –650 | +433 | +217 |
| (c) | +200 | +200 | +200 |

## Chapter 4

**4.1** –40.0 MPa; $-571\times10^{-6}$; –686 μm

**4.3** 57.6 MPa, –40.7 MPa; 1.16 mm, –0.582 mm = –582 μm

**4.5** 9.00 ksi, 3.00 ksi; +0.0540 in., +0.0120 in.; 0.0540 in. down, 0.0660 in. down

**4.7** –957 psi; $-6.12\times10^{-3}$ in.

**4.9** 6.43 kips left; 8.57 kips left; 0.00343 in. right

**4.11** $-FL/(2AE)$; plot; $-FL/(3AE)$

**4.13** $\dfrac{-2}{\pi(0.4+x)^2}$ MPa, $x$ in meters ; –34.1 μm

**4.15** 582 μm right, 582 μm up

**4.17** 56.6 MPa, –56.6 MPa; 4.2; +1.143 mm, –1.143 mm; 1.616 mm right, 0 mm

**4.19** 80 MPa, –80 MPa; 3.0; +2.29 mm, –2.29 mm; 4.58 mm right

**4.21** $\sigma_{AB}$ = 56.6 MPa, $\sigma_{BC}$ = –56.6 MPa, $\sigma_{BD}$ = 0 MPa; 4.2; $\Delta_{AB}$ = +1.143 mm; $\Delta_{BC}$ = –1.143 mm; $\Delta_{BD}$ = 0 mm; 1.616 mm right

**4.23** $\sigma_s$ = –4.80 ksi; $\sigma_c$ = –0.640 ksi; –0.0230 in.; 8330 ksi

**4.25** 0.214 in$^2$; 0.0672 in.

**4.27** –40.3 MPa

**4.29** 11.43 ksi, 8.57 ksi; 317°F

**4.31** 392 MPa (matrix), –727 MPa

**4.33** 40.0 MPa; 5.00 MPa; $140\times10^{-6}$

**4.35** 7.5 mm minimum; 18.3 mm

**4.37** 589 lb; $-82.5\times10^{-6}$; $250\times10^{-6}$

**4.39** 3.38 MPa; 62.8 kN/m

**4.41** 120 kips

**4.43** $3\sigma_{ave}$; 2.67$a$ ($n$ = 2.67 radii)

**4.45** 64.6 N·m, 22.9 N·m; 8.75 mm down; 2.29 MN/m

**4.47** 8.0 N·m, 22.6 N·m; 3.06 mm right; 6.53 MN/m

**4.49** $3FL/(AE)$ down; $EA/(3L)$; $m = 9K(\rho/E)L^2$; minimize $\rho/E$ or maximize $E/\rho$

**4.51** $U_{AB} = U_{BC} = (3EAv^2)/(16L)$,
$U_{BD} = (EAv^2)/(2\sqrt{3}L)$;
$4.19\times10^{-3}$ in.; 1195 kips/in.

**4.53** 4.40 N·m; 0.220 mm down

**4.55** 25.4 lb-in.; 0.0254 in. down

**4.57** $P_{AD} = 100.7$ kN, $P_{BD} = 112.0$ kN, $P_{CD} = -28.0$ kN; 107.0 kN right, 142.6 kN down; 168.2 kN

## Chapter 5

**5.1** 25.1 in$^4$; 3.98 ksi; 5.0; 0.622°

**5.3** $5.796\times10^{-6}$m$^4$; $5.726\times10^{-6}$m$^4$; –1.2%

**5.5** 45.7 mm; 11.44 mm, 45.8 mm; 62.0 mm, 6.20 mm; 6.0%, 63.2% savings

**5.7** 10.2 ksi; 2.95°

**5.9** 2.03 ksi, 1.510 ksi; 0.716°

**5.11** 15.71 kip-in.; 2.74 in., 3.0 in.; 63% savings

**5.13** 174.3 N·m; 0.130°

**5.15** 1.273 ksi; 1.12° (approx.)

**5.17** 1.48 in.

**5.19** 14.3 kN·m; 73.0 MPa; 3.34°; 128 mm

**5.21** 3183 N·m; 32.3 mm; 41.9 mm; 41.9 mm

## Chapter 6

**6.1** (a) $0 < x < 4$ m: $V(x) = -6.0$ kN,
$M(x) = 6x$ kN·m;
$4 < x < 10$ m: $V(x) = +4.0$ kN;
$M(x) = -4x+40$ kN·m;
(b) $V_{max} = 6$ kN; $M_{max} = 24$ kN·m at $x = 4$ m

**6.3** 22.9 in.; 2.18%

**6.5** S12×31, W14×26; W14×26

**6.7** 72 kip-in.; 6.00 ksi; 10.39 in.

**6.9** 480 MPa

**6.11** 4.10 ksi at $x = 21$ ft

**6.13** 794 psi

**6.15** 105.6 MPa; 2.27

**6.17** $M = \frac{-w_o}{6L}(L-x)^3$; 29.4 ksi

**6.19** 24.9 mm

**6.21** 21.2 MPa; 2.47 mm difference

**6.23** $v = \frac{P}{6EI}(x^3 - 3Lx^2)$; beam hits structure

**6.25** $L = a+b$; answers will vary based on length variables ($a$, $b$, $L$) used.

$$v_1 = \frac{-Pbx}{6EIL}[L^2 - b^2 - x^2]; \quad 0 < x < a$$

$$v_2 = \frac{-P}{6EIL}[ax^3 - 3aLx^2 + (3a^2L - bL^2 + b^3)x - a^3L]; \quad a < x < a+b = L$$

If $a < b$:

$$\delta_{max} = \frac{-Pb(L^2-b^2)^{3/2}}{9\sqrt{3}\,EIL} \quad \text{at} \quad x = \sqrt{\frac{L^2-b^2}{3}}$$

**6.27**

$$v_1 = \frac{P}{6EI}(x^3 - 3Lx^2); \quad 0 < x < L$$

$$v_2 = \frac{-PL^2}{6EI}(3x - 2L); \quad L < x < 3L/2$$

**6.29**

$$v_1 = \frac{P}{6EI}[x^3 - 3a(a+s)x]; \quad 0 < x < a$$

$$v_2 = \frac{P}{6EI}[3ax^2 - 3a(2a+s)x + a^3]; \quad a < x < a+s$$

**6.31** $R_A = 3wL/8$, $R_B = 5wL/8$,
$M_B = wL^2/8$ CW;
$v = \frac{-w}{48EI}[2x^4 - 3Lx^3 + L^3x]$;
$v(L/2) = \frac{-wL^4}{192EI}$

**6.33** $R_A = R_B = wL/2$ up,
$M_A = wL^2/12$ CCW,
$M_B = wL^2/12$ CW;
$\delta_{max} = -wL^4/(384EI)$

**6.35** $\sigma = \pm12$ MPa at $(2, \mp0.05)$m
$\tau = \pm0.3$ MPa at $(2 \pm 2, 0)$m
$\delta_{max} = 2.00$ mm down

**6.37** 15.2 kN; 59.2 MPa

**6.39** 333 kPa; 533 kPa; 416 kPa

**6.41** 274 MPa; 2.39 MPa

**6.43** 6.1 in.

**6.45** 42.7 $in^3$; 21.3 $in^3$; 6.28 $in^3$

**6.47** $AD/2$

**6.49** 13.8

**6.51** 80×240 mm; S12×31.8; 68.6% or 69.4 lb/ft savings

**6.53** 333 lb, deflection

**6.55** Silicon Carbide (SiC), but it is brittle; Wood is next best

**6.57**

$$m = \left(\frac{5\pi}{96}\right)^{1/2}\left(\frac{wL}{\delta}\right)^{1/2}\left(\frac{\rho^2}{E}\right)^{1/2}\left(\frac{1}{\phi}\right)^{1/2}L^{5/2};$$

$$\phi_{st} = 64.7;\ \phi_{al} = 22.2$$

## Chapter 7

**7.1** –89.1 MPa; –2.60 MPa

**7.3** at $A$: $\sigma = +142.9$ psi; $B$: $\sigma = -28.6$ psi

**7.5** at $A$: $\sigma_y = 5728$ psi; $B$: $\sigma_y = -5731$ psi; $C$: $\sigma_y = -1.6$ MPa, $\tau_{xy} = -32.$ psi
*Note*: $\sigma_a = -1.6$ psi (at $A$, $B$, $C$); $\sigma_b = 5729.6$ psi (+ at $A$, – at $B$)

**7.7** at $A$: +6.67 MPa; $B$: –1.33 MPa; $C$: –9.33 MPa; $D$: –1.33 MPa

**7.9** –2.39 MPa; 1.90 MPa

**7.11** $R/4$

**7.13** $4Pd/(3L^2)$

**7.15** $\sigma_b = 82.8$ MPa (+ at $B$);
$\tau_T = 6.90$ MPa (+ at $A$, $B$; – at $C$);
$\tau_V = 0.28$ MPa (– at $A$, $C$)

**7.17** $\sigma_a = P/A = 67.9$ psi (– at $A$, $B$, $C$);
$\sigma_{b,x} = FeD/(2I) = 1207$ psi (+ at $B$);
$\sigma_{b,y} = PhD/(2I) = 2897$ psi (+ at $A$, – at $C$);
$\tau_T = FhD/(2J) = 966$ psi (+ at $A$, $B$; – at $C$);
$\tau_V = 4F/(3A) = 60.4$ psi (– at $A$, $C$)

**7.19**

$$v = \frac{-wx}{24EI}[L_1^3 - 2L_1x^2 + x^3] - \frac{wL_2x}{2AE};$$

$\delta_{max} = -0.820$ in. down at $x = 121.25$ in.

**7.21** $$v = \frac{-wx}{48EI}[-2x^3 + 3Lx^2 - L^3],$$

$$v' = \frac{-w}{48EI}[-8x^3 + 9Lx^2 - L^3];$$

$$A_y = R = \frac{3wL}{8},\ B_y = \frac{5wL}{8};$$

$$M_B = \frac{wL^2}{8}\ \text{CW}$$

## Chapter 8

**8.1** (a) 188.3 MPa, 111.7 MPa, –32.1 MPa
(c) 244.3 MPa, 55.7 MPa, –3.3 MPa
(e) 176.5 MPa, –76.5 MPa, –113.6 MPa
(g) 242.3 MPa, 57.7 MPa, 19.6 MPa
(i) –179.1 MPa, –120.9 MPa, 89.7 MPa

**8.3** (a) 17.1 ksi@–19.3°, –2.1 ksi@70.7°
(c) 16.4 ksi@12.8°, –11.4 ksi@102.8°
(e) 19.0 ksi@–33.7°, 6.0 ksi@56.3°
(g) 11.7 ksi@–15.5°, –11.7 ksi@74.5°

**8.4** (a) 9.6 ksi@115.7°,–9.6 ksi@25.7°, 7.5 ksi
(c) 13.9 ksi@–32.3°,–13.9 ksi@57.8°, 2.5 ksi
(e) 6.5 ksi@101.3°,–6.5 ksi@11.3°, 12.5 ksi
(g) 11.7 ksi@119.5°,–11.7 ksi@29.5°, 0 ksi

**8.5** 45.0 MPa, –26.0 MPa; 30.0 MPa

**8.7** $\sigma = -2.60$ MPa, $\tau = -89.09$ MPa;
$\sigma_{I,II} = +87.8$ MPa, –90.4 MPa;
$\tau_{max} = 89.14$ MPa

**8.9** 46.4 MPa; 0 MPa; 0 MPa, –42.8 MPa

**8.11** 1.74 ksi @ Pt. $B$; 1.14 ksi @ Pt. $B$.

**8.12** (a) 7.7 ksi, –7.70 ksi, 0.885 ksi
(b) 15.4 ksi, –15.4 ksi, 4.74 ksi
(c) 15.0 ksi, –15.8 ksi, 0 ksi
(d) 16.8 ksi, –17.8 ksi, 0 ksi (normal stresses along $y$-axis, at points not on $x$- or $z$- axes)
(e) 3.85 ksi, 8.61 ksi, 7.9 ksi, 8.88 ksi, corresponding to (a)-(d)

**8.13** (a) 14.42 ksi, 10.58 ksi, –1.61 ksi
(c) 3.55 ksi, 1.45 ksi, 13.83 ksi

(e) 141.0 MPa, 59.0 MPa, 17.8 MPa

(g) 70.9 MPa, –30.9 MPa, –94.9 MPa

**8.14** *Note*: $\theta_s$ defines plane where $\tau_{max} > 0$.

(a) 15 ksi@0°, 10 ksi@90°; $\tau_{max} = 2.5$ ksi, $\sigma_{ave} = 12.5$ ksi, $\theta_s = -45°$

(c) 16.4 ksi@12.8°, –11.4 ksi@102.8°; $\tau_{max} = 13.9$ ksi, $\sigma_{ave} = 2.5$ ksi, $\theta_s = -32.2°$

(e) 144.7 MPa@31.7°, 55.3 MPa@121.7°; $\tau_{max} = 44.7$ MPa, $\sigma_{ave} = 100$ MPa, $\theta_s = -13.3°$

(g) 127.7 MPa@–10.9°, –87.7 MPa@79.1°; $\tau_{max} = 107.7$ MPa, $\sigma_{ave} = 20$ MPa, $\theta_s = -55.9°$

**8.15** (a) $169.9\times10^{-6}$, $80.1\times10^{-6}$, $29.1\times10^{-6}$

(c) $71.4\times10^{-6}$, $178.6\times10^{-6}$, $-31.9\times10^{-6}$

**8.16** (a) $172.2\times10^{-6}$@29.0°, $77.8\times10^{-6}$@119.0°; $94.3\times10^{-6}$, 7.08 MPa

**8.17** 121 GPa, 0.28

**8.18** (a) $208\times10^{-6}$, $91.7\times10^{-6}$, $116.6\times10^{-6}$

**8.19** $\varepsilon_x = \varepsilon_A$, $\varepsilon_y = \frac{1}{3}[-\varepsilon_A + 2\varepsilon_B + 2\varepsilon_C]$,

$$\gamma_{xy} = \frac{2}{\sqrt{3}}[\varepsilon_B - \varepsilon_C]$$

**8.20** (a) 5.0 ksi; 7.5 ksi; I-III (out-of-plane)

(c) 2.5 ksi; 3.5 ksi; I-III (out-of-plane)

## Chapter 9

**9.1** Tresca or von Mises; Maximum Normal Stress

**9.3** $\sigma_I = 3.1$ ksi, does not fail

Try: $T = 120$ lb and $M = 160$ lb-ft ($\sigma_I = 14.4$ ksi)

**9.5**

$$U_D = \frac{1}{2E}\Big[\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - 2\nu(\sigma_I\sigma_{II} + \sigma_{II}\sigma_{III} + \sigma_{III}\sigma_I)\Big]$$

$$U_D = \frac{1}{2E}\Big[\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - (\sigma_I\sigma_{II} + \sigma_{II}\sigma_{III} + \sigma_{III}\sigma_I)\Big]$$

$$\sigma_o = \left[\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}{2}\right]^{1/2}$$

## Chapter 10

**10.1** 140 in. = 11.7 ft

**10.3** 395 kN, *AE* and *CD*

**10.5** 106.1 in. = 8.84 ft

**10.7** (a) 708 kips ($y$-axis buckling); 760 kips ($z$-axis buckling); buckling about weak-axis;

(b) 1390 kips ($y$-axis buckling); 760 kips ($z$-axis buckling); buckling about strong-axis

**10.9** $\dfrac{K_T\theta}{L\sin\theta + e\cos\theta}$; plot

**10.11** 0.36 μm

**10.13** $m = \dfrac{2}{\pi^2}P_{cr}\left(\dfrac{\rho}{E}\right)\dfrac{L^3}{R^2}$;

1st: Ceramic, 2nd: Composite